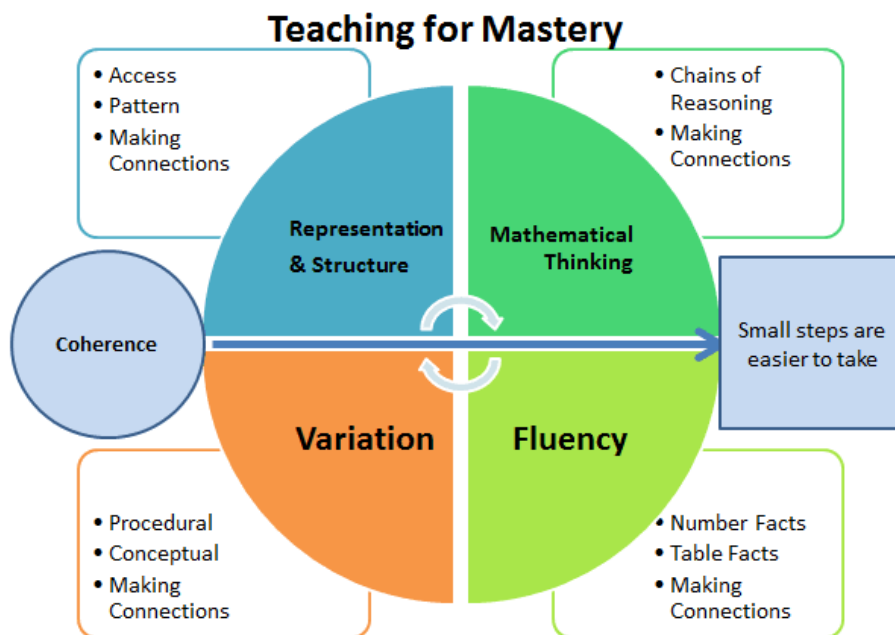
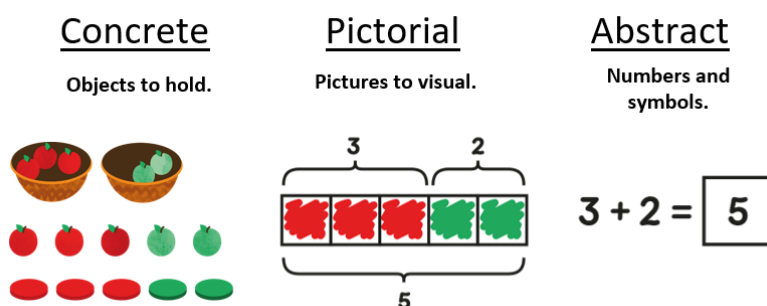


Calculation Policy Updated 2020

At Glebelands, we have adopted the mastery approach to mathematics. This means that our lessons will focus on helping the children to become fluent in their calculations, confident in their understanding of number and place value and apply this through reasoning and problem solving.



Our calculation policy is structured using the mastery approach focusing on the three main elements: concrete, pictorial and abstract.



The children need to use manipulatives (such as place value counters, multi-link, Dienes, Numicon, number beads and Cuisenaire rods) to fully understand number and place value. This will lead onto pictorial representations where children use drawings or models to deepen their understanding of concepts. Finally, they will use more abstract methods focusing more on number representations.

It is essential that children deepen their understanding of concepts within their year group and apply these to more complex problems. They should be given the opportunity to find patterns in numbers and make links between concepts. The teaching of number should be intertwined with other areas of mathematics such as ratio, algebra, measures, fractions, decimals, percentages and money.

Aims

Foundation Stage

In Foundation Stage, we follow the Early Learning Goals.

In Maths, these are split into two categories:

- Number
- Numerical Patterns

Children need to have a deep understanding of numbers from 1-10, recognising number bonds to at least 5 and finding patterns within the numbers. This needs to be applied to many different contexts to allow the children to subitise (recognise without counting) to at least 5.

Key Stage 1

In KS1, the children need to gain confidence and mental fluency with whole numbers, counting and place value.

Addition and subtraction

Children in KS1 should develop their fluency in mental methods such as bridging to the nearest 10 and understanding the rules of commutativity. They should use a variety of models and images to add and subtract with increasingly larger numbers, begin to use related facts and understand how to use symbols such as $+$ $-$ $<$ $>$. Written methods should begin with a focus on part/ part/ whole and move onto more formal methods of number lines and column method.

Multiplication and division

Children in KS1 should link multiplication to addition with the concept of repeated addition. They should learn about grouping and sharing for division and begin to use more formal methods such as arrays. They should extend their understanding of doubling and halving from Foundation Stage with numbers beyond 20. They will begin to use symbols such as \times \div and will begin to use terms like commutative to show the relationship between numbers.

Times Tables

By the end of Year 2, children are expected to know and recall:

- 2 times tables
- 10 times tables
- 5 times tables

Lower Key Stage 2

In Years 3 and 4, children should become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value.

Addition and subtraction

Children in Years 3 and 4 should increase their confidence with mental calculations and develop efficient written methods. They need to perform calculations accurately, checking their answers using the inverse operation.

Multiplication and division

Children in Years 3 and 4 should develop their knowledge of mental calculations using related facts, commutativity and distributive law. They should develop their confidence with using arrays for multiplication and begin to use this for division. They should extend this to using more formal methods for multiplication (grid method and short multiplication) and division (short division). They will be introduced to the idea of remainders.

Times Tables

By the end of Year 3, children are expected to know and recall all of the times tables from Year 2 along with:

- 3 times tables
- 4 times tables
- 8 times tables

By the end of Year 4, children are expected to know and recall all of the times tables from Year 2 and Year 3 along with:

- 6 times tables
- 7 times tables
- 9 times tables
- 11 times tables
- 12 times tables

Upper Key Stage 2

In Years 5 and 6, children should extend their understanding of number and place value. Methods will become more complex and there will be greater emphasis on problem solving and reasoning.

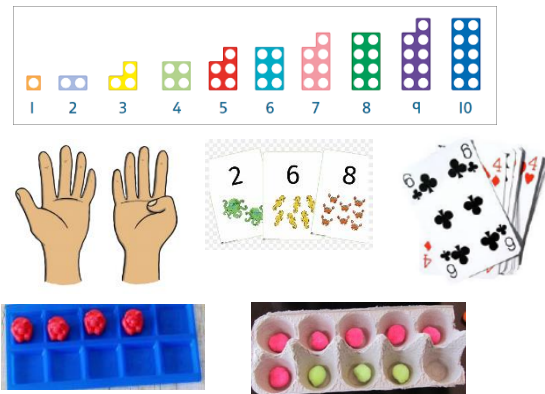
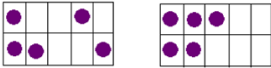



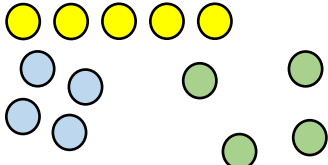
Addition and subtraction

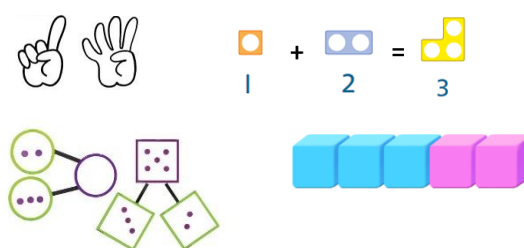
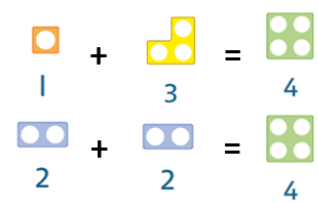

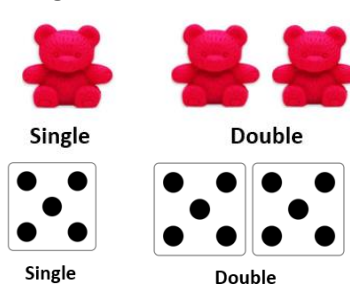
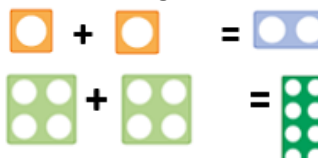







Children in Years 5 and 6 should develop their ability to solve a wider range of problems, including increasingly complex properties of number and arithmetic. They need to choose the most efficient methods for mental and written calculations.

Multiplication and division




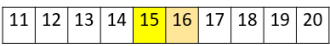


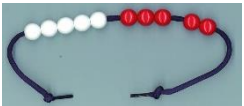

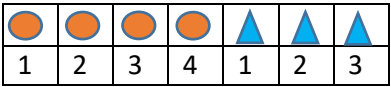
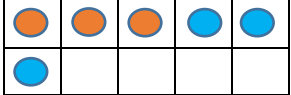
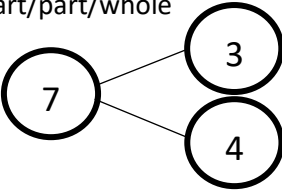
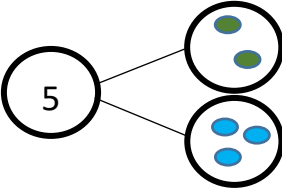

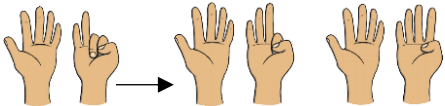
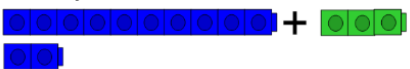
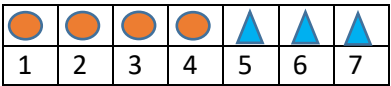
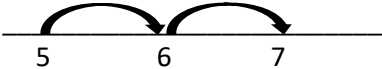
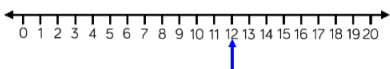
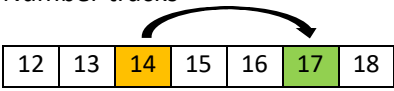
Children in Years 5 and 6 should develop their confidence with mental calculations, using increasingly larger numbers and decimals. Written methods should become fully integrated: short and long multiplication (including decimals) and short and long division. Children should select the most efficient methods for calculations, checking these using the inverse operation. Remainders should be interpreted using decimals, fractions and rounding.

Addition and Subtraction

Foundation Stage			
Outcome	Concrete	Pictorial	Abstract
Know numbers to 10	<p>Using familiar items to count to 10 and recognise up to 10 items.</p> 	<p>Tens frames</p>  <p>Numicon images</p> 	<p>Number formation</p> 
Subitise (know without counting) numbers to 5	<p>Need to subitise numbers to 5.</p> 	<p>Begin with linear representations, then circular then scrambled.</p> 	

<p>Numbers bonds to 5</p>	<p>Using familiar items to create numbers up to 5 in lots of different ways.</p> 	<p>Numicon images</p>  <p>Five frames</p> 	
<p>Know some double facts to 10</p>	<p>Using familiar items to double numbers</p> 	<p>Numicon images</p>  <p>Tens frames</p> 	
<p>Compare quantities to 10</p>	<p>Using familiar items to identify quantities. Consider which has the greater amount, fewer or the same quantities.</p> 	<p>Identify quantities to 10 from images.</p> 	<p>Match up numbers to quantities.</p>
<p>Identify patterns</p>	<p>Using familiar objects to create or identify patterns in number and sorting objects into equal groups.</p> 	<p>Explore patterns in drawings and images. Looking at positional images, e.g. first, second, third, etc.</p>  <p>Look at patterns in size, shape, colour, position or quantity.</p>  <p>Odd one out</p> 	<p>Focus on odd or even numbers.</p>

Year 1

Outcome	Concrete	Pictorial	Abstract
<p>Identify one more or one less of a given number.</p>	<p>Counting on and back using familiar objects and resources.</p> <p>One more </p> <p>One less </p>	<p>Introduce tens frames and number tracks.</p>  <p>One more than 15</p> 	<p>Introduction to + - and = symbols to create number sentences.</p> <p>$5 - 1 = 4$ $4 + 1 = 5$</p>
<p>Use addition to combine groups.</p>	<p>Counting using familiar objects and resources.</p>  <p>Count objects</p>  <p>1, 2, 3, 4 1, 2, 3</p> <p>1, 2, 3, 4, 5, 6, 7</p>  <p>Using Cuisenaires (links to bar modelling)</p> 	<p>Bar models</p>  <p>7</p> <p>Tens frame</p>  <p>Part/part/whole</p>  	<p>Using number sentences and beginning to calculate mentally.</p> <p>$7 + 2 = 9$ $2 + 7 = 9$ $9 = 2 + 7$ $9 = 7 + 2$</p>
<p>Use addition to count on.</p>	<p>I have 3 sweets. I buy 2 more. How many do I have?</p>  <p>Hold the number in your head and count on.</p> <p>7 add 2</p>  <p>Start at 7 Count up to 9</p> <p>Use objects to count on</p> 	<p>Bar models</p>  <p>Empty number lines</p>  <p>Number lines</p>  <p>Number tracks</p> 	<p>Missing number problems.</p> <p>$8 = 3 + \square$ $9 = \square + 2$ $9 = \square + 5$ $4 = 9 - \square$ $\square = 9 - 4$</p>

Doubling numbers within 20

Use objects and manipulatives.

Part/part/whole (link to halving)

Using number sentences and beginning to calculate mentally.
 $6 + 6 =$
 Double 9 =
 $14 = \text{Double } \dots$
 Half of 18 = ...
 $10 = \text{half of } \dots$
 $7 = 14 - \square$
 $4 + \square = 8$

Recall and use addition and subtraction facts to 20 **fluently** aiming to select the most efficient method.

Addition and subtraction facts

Bar models

Total	
Part	Part

12	
5	7

Empty number lines

Jumping in ones

Jumping in ones and tens

Using number sentences and calculating mentally.
 $13 + 4 = 17$
 $17 = 13 + 4$
 $4 + 13 = 17$
 $17 = 4 + 13$
 $17 - 4 = 13$
 $13 = 17 - 4$
 $17 - 13 = 4$
 $4 = 17 - 13$

Know how to use add (+), subtract (-) equals (=) signs.


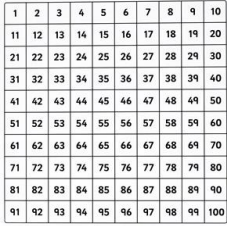



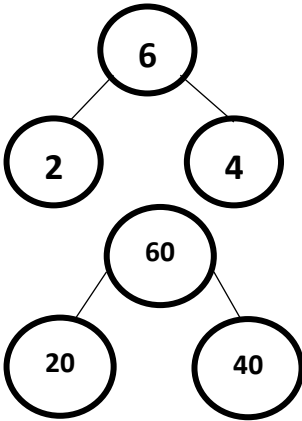
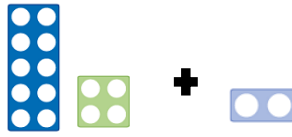
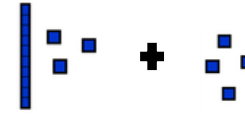
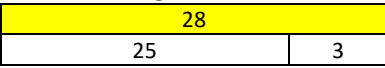
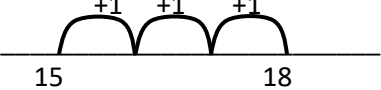
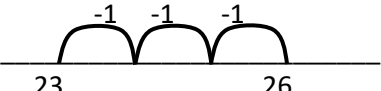
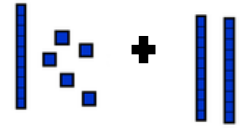

Use of manipulatives: Number cards

Dienes

Use of part/ part/ whole to understand commutative rule.

Missing number problems.
 $7 = \square - 9$
 $\square + 4 = 11$

Year 2

Outcome	Concrete	Pictorial	Abstract															
Compare and use numbers to 100 using < > =	Use familiar objects to compare value of numbers: 	Use 100 squares: 	Using number sentences using < > = 15 > 11 41 < 60 16 = 8 + 8															
Derive and use related facts to 100	Use familiar objects: Number beads to 100   $3 + 7 = 10$  $30 + 70 = 100$	Using part/ part/ whole: 	Using number sentences and calculating mentally. $20 + 80 = 100$ $100 - 80 = 20$ $80 + 20 = 100$ $100 - 20 = 80$ $100 = 20 + 80$ $80 = 100 - 20$ $100 = 80 + 20$ $20 = 100 - 80$ Missing number problems: $70 = 100 - \square$ $\square + 40 = 100$															
TU ± O (without crossing tens boundary)	Using familiar objects:  $14 + 2 =$  $13 + 4 =$	Bar modelling:  Number lines:  	Number sentences: $34 + 3 = 37$ $67 = 61 + 6$															
TO ± T	Dienes:  $15 + 20 =$	Understand place value: <u>PV chart:</u> $43 + 20 =$ <table border="1" data-bbox="708 1563 1007 1749"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td>4</td> <td>3</td> </tr> <tr> <td></td> <td>+2</td> <td></td> </tr> <tr> <td></td> <td>↓</td> <td></td> </tr> <tr> <td></td> <td>6</td> <td>3</td> </tr> </tbody> </table> Changing the 'tens' only <u>Hundreds square:</u> $45 + 30 =$ 	Hundreds	Tens	Ones		4	3		+2			↓			6	3	Number sentences: $34 + 30 = 64$ $82 = 50 + 32$ Missing number problems: $44 = 24 + \square$ $\square + 40 = 72$
Hundreds	Tens	Ones																
	4	3																
	+2																	
	↓																	
	6	3																

TO ± TO
(without crossing tens boundary)

Dienes:



$$23 + 15$$



$$25 - 13$$

Including finding the difference:



$$24 - 13$$

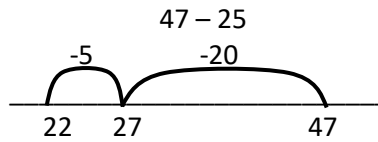
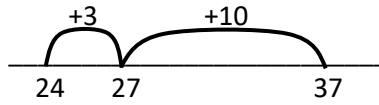
Place value counters:

$$26 + 13 =$$

Tens	Ones

Number lines to support mental calculations (always start with ones to link with column method):

$$24 + 13$$



Moving onto expanded column:

$$\begin{array}{r} 20 + 6 \\ + 10 + 3 \\ \hline 30 + 9 = 39 \end{array}$$

$$\begin{array}{r} 40 + 8 \\ - 20 + 5 \\ \hline 20 + 3 = 23 \end{array}$$

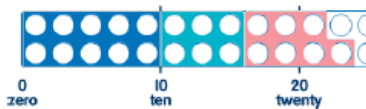
Compact column method:

$$\begin{array}{r} 26 \\ + 13 \\ \hline 39 \end{array}$$

$$\begin{array}{r} 48 \\ - 25 \\ \hline 23 \end{array}$$

TO ± TO
(crossing the tens boundary using bridging)

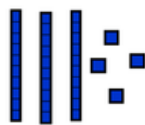
Numicon:



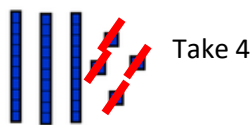
$$16 + 7 \text{ becomes } 16 + 4 + 3$$

Dienes:

$$34 - 16 =$$



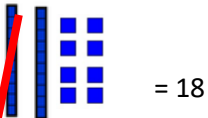
Partition 16 into 10 + 4 + 2



Change ten into 10 ones



Now subtract 10



$$= 18$$

Number lines to bridge to nearest 10:

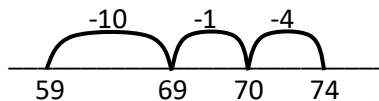
$$74 - 15$$

Know how to partition numbers in many different ways:

E.g. Some examples of 15 are:

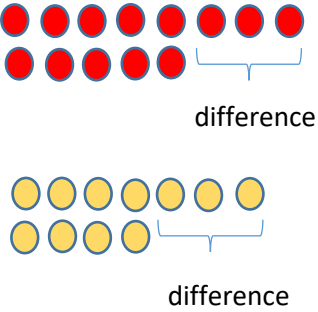
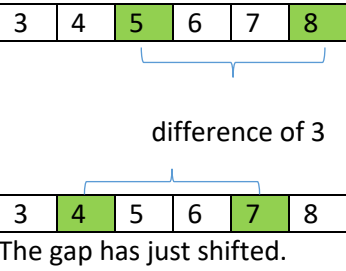

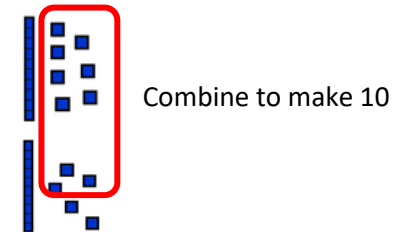
$$\begin{array}{lll} 10 + 5 & 10 + 1 + 4 & 11 + 2 + 3 \\ 9 + 6 & 9 + 4 + 2 & 5 + 5 + 5 \end{array}$$

Partition 15 into 10 + 4 + 1



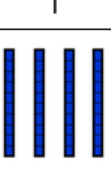

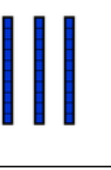

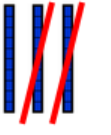

Bridge to nearest 10 then subtract the rest. Always start with the ones to link to column method.

Year 3

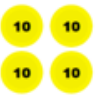




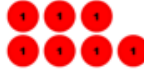
Outcome	Concrete	Pictorial	Abstract						
<p>Adjust numbers to subtract mentally</p>	<p>Show the ideas behind the adjustment by linking to finding the difference.</p> <p>$8 - 5$ is the same as $7 - 4$</p> 		<p>Adjustment for subtraction of multiples of 10.</p> <p>E.g. $80 - 46$</p> <p>Adjust by shifting 1</p> <p>80 becomes 79</p> <p>46 becomes 45</p> <p>$79 - 45 = 34$</p>						
<p>TO + TO (keeping new tens with the ones)</p>	<p>Dienes:</p> <p>$27 + 14$</p> <p>T O</p>  <p>Place value counters:</p> <p>$27 + 37 =$</p> <table border="1" data-bbox="308 1043 592 1261"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table>	Tens	Ones					<p>Expanded method:</p> <p>$58 + 34$</p> <p>T O</p> <p>$50 + 8$</p> <p>$30 + 4$</p> <p>$80 + 12 = 92$</p> <p>Always start by adding the ones then move across to the tens. At this point, keep the tens in the ones column to understand the concepts.</p>	
Tens	Ones								
<p>TO + TO (crossing boundary and adjusting)</p>	<p>Dienes:</p> <p>$17 + 15 =$</p> <p>T O</p>  <p>Place value counters:</p> <p>$28 + 37 =$</p> <table border="1" data-bbox="308 1742 643 1995"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table> <p>Combine the units to make a ten then exchange this over to the tens column.</p>	Tens	Ones					<p>Expanded column crossing the tens boundary.</p> <p>$47 + 18$</p> <p>T O</p> <p>$40 + 7$</p> <p>$+ 10 + 8$</p> <hr/> <p>$60 + 5 = 65$</p> <p>10</p> <p>↑</p> <p>Add the ones first</p> <p>Show the extra 10 in the tens column.</p>	<p>Compact column method.</p> <p>$47 + 18$</p> <p>T O</p> <p>4 7</p> <p>$+ 1 8$</p> <hr/> <p>6 5</p> <p>1</p> <p>↑</p> <p>Add the ones first.</p> <p>This time show the extra tens in the compact form.</p>
Tens	Ones								

TO – TO
(crossing the
tens
boundary)

Dienes
43 - 26

T	O	
		Start by subtracting the ones you can
		Exchange a ten to make 10 ones. Then subtract the remaining ones.
		Finally, subtract the tens.

Place value counters
Same as above

T	O
	
	
	

Expanded column method
with exchange

$$43 - 26$$

$$\begin{array}{r} \text{T} \quad \text{O} \\ 30 \\ \cancel{40} + 13 \\ - \quad 20 + 6 \\ \hline 10 + 7 = 17 \end{array}$$

Exchange a ten into the ones column when there are not enough ones for the calculation.

Compact column method.

$$43 - 26$$

$$\begin{array}{r} \text{T} \quad \text{O} \\ 3 \\ \cancel{4} 13 \\ - \quad 26 \\ \hline 17 \end{array}$$

HTO ± TO
HTO ± HTO

Use the same manipulatives as above. Ensure at this stage children only cross one set of boundaries (hundreds OR tens)

Expanded method
156 – 38

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 100 + \cancel{50} + 16 \\ - \quad \quad 30 + 8 \\ \hline 100 + 10 + 8 = 118 \end{array}$$

$$352 + 165$$

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 300 + 50 + 2 \\ + 100 + 60 + 5 \\ \hline 500 + 10 + 7 = 517 \\ 100 \end{array}$$

Compact method
156 – 38


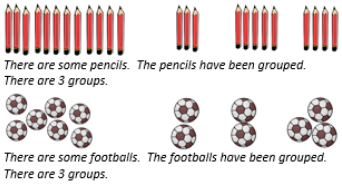

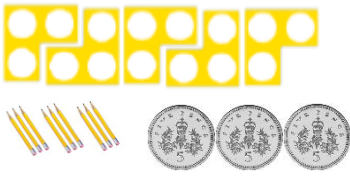
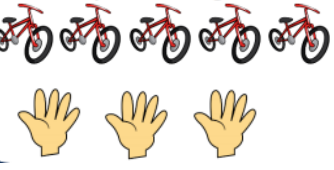

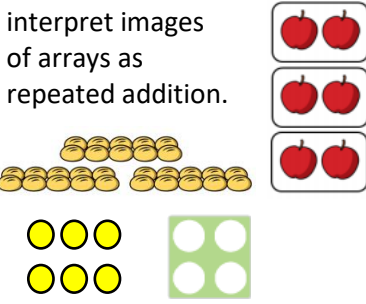

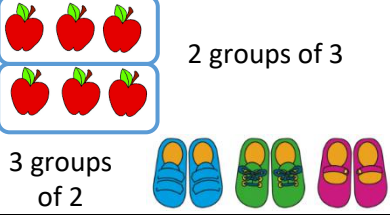

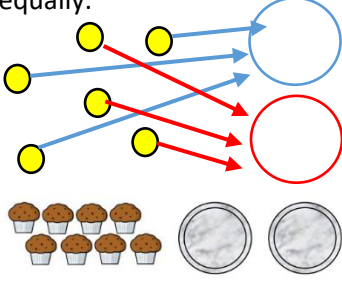
$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 4 \\ 1 \cancel{5} 16 \\ - \quad 38 \\ \hline 118 \end{array}$$



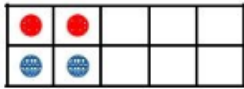
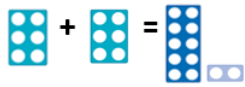
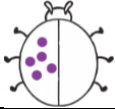
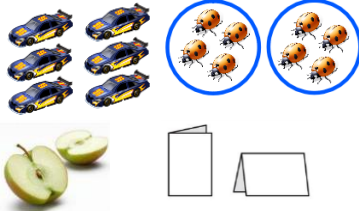
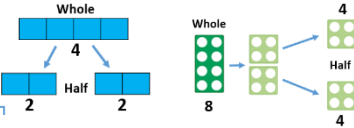
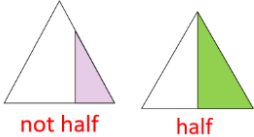

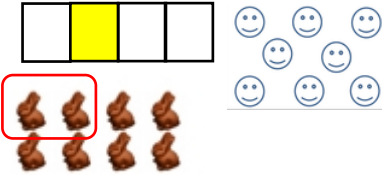
$$352 + 165$$



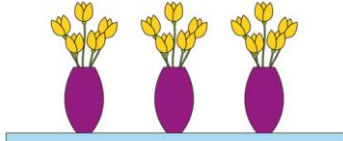

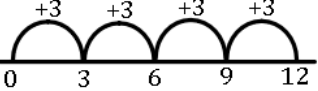
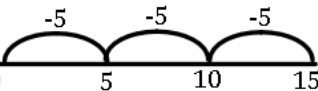
$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 3 \quad 5 \quad 2 \\ + 1 \quad 6 \quad 5 \\ \hline 5 \quad 1 \quad 7 \\ 1 \end{array}$$



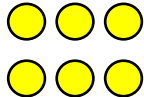
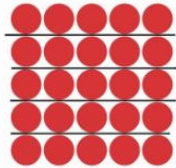
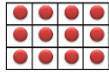



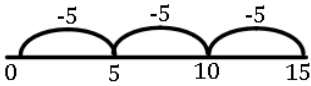
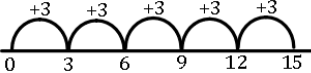
Year 4																																																											
Outcome	Concrete	Pictorial	Abstract																																																								
Difference with close numbers	Use bridging from Year 2 for manipulatives.	<p>Question: 304 - 295</p> <p>Step 1: Decide whether the numbers are close together.</p> <p>Step 2: Bridge to the nearest 10 or 100 using a number line</p> <p>Step 3: Add or subtract the remaining numbers.</p>																																																									
HTO ± HTO ThHTO ± ThHTO	Use the methods from Year 3. Progression is crossing more than one boundary.																																																										
HTO ± HTO ThHTO ± ThHTO (exchanging through zero)	Place value counters and Dienes to show that 10 tens make 100, 10 hundreds make 1000.	Expanded method $304 - 137$ <table style="margin-left: auto; margin-right: auto;"> <tr><td>H</td><td>T</td><td>O</td></tr> <tr><td>200</td><td>90</td><td></td></tr> <tr><td>300</td><td>+ 100</td><td>+ 14</td></tr> <tr><td>-</td><td>100</td><td>+ 30 + 7</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>200</td><td>+ 60</td><td>+ 7 = 267</td></tr> </table>	H	T	O	200	90		300	+ 100	+ 14	-	100	+ 30 + 7	<hr/>			200	+ 60	+ 7 = 267	Compact method $304 - 137$ <table style="margin-left: auto; margin-right: auto;"> <tr><td>H</td><td>T</td><td>O</td></tr> <tr><td>2</td><td>9</td><td></td></tr> <tr><td>3</td><td>10</td><td>14</td></tr> <tr><td>-</td><td>1</td><td>3 7</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>1</td><td>6</td><td>7</td></tr> </table>	H	T	O	2	9		3	10	14	-	1	3 7	<hr/>			1	6	7																				
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Decimals O.t ± O.t O.th ± O.th	Place value counters <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Ones</td> <td>•</td> <td>Tenths</td> <td>Hundredths</td> </tr> <tr> <td></td> <td>•</td> <td></td> <td></td> </tr> </table> <p>Progression:</p> <ul style="list-style-type: none"> - addition without boundaries - addition with exchanging - subtraction without boundaries - subtraction with one exchange - subtraction with more than one exchange - subtraction with exchanges through zero 	Ones	•	Tenths	Hundredths		•			Expanded method $2.5 + 1.7$ <table style="margin-left: auto; margin-right: auto;"> <tr><td>O</td><td>t</td></tr> <tr><td>2</td><td>+ 0.5</td></tr> <tr><td>+</td><td>1 + 0.7</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>4</td><td>+ 0.2 = 4.2</td></tr> <tr><td>1</td><td></td></tr> </table> $3.4 - 1.6$ <table style="margin-left: auto; margin-right: auto;"> <tr><td>O</td><td>t</td></tr> <tr><td>2</td><td>1.4</td></tr> <tr><td>3</td><td>+ 0.4</td></tr> <tr><td>-</td><td>1 + 0.6</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>1</td><td>+ 0.8 = 1.8</td></tr> </table>	O	t	2	+ 0.5	+	1 + 0.7	<hr/>		4	+ 0.2 = 4.2	1		O	t	2	1.4	3	+ 0.4	-	1 + 0.6	<hr/>		1	+ 0.8 = 1.8	Compact method $2.5 + 1.7$ <table style="margin-left: auto; margin-right: auto;"> <tr><td>O</td><td>t</td></tr> <tr><td>2</td><td>. 5</td></tr> <tr><td>+</td><td>1 . 7</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>4</td><td>. 2</td></tr> <tr><td>1</td><td></td></tr> </table> $3.4 - 1.6$ <table style="margin-left: auto; margin-right: auto;"> <tr><td>O</td><td>t</td></tr> <tr><td>2</td><td></td></tr> <tr><td>3</td><td>. 14</td></tr> <tr><td>-</td><td>1 . 6</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>1</td><td>. 8</td></tr> </table>	O	t	2	. 5	+	1 . 7	<hr/>		4	. 2	1		O	t	2		3	. 14	-	1 . 6	<hr/>		1	. 8
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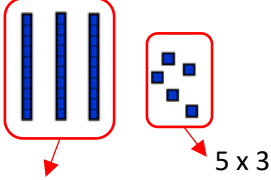












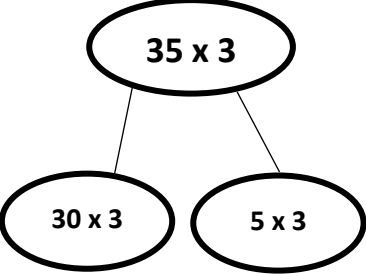






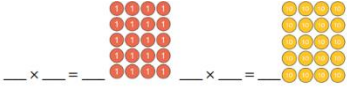

Multiplication and Division

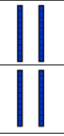

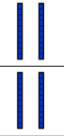









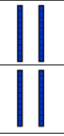

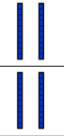









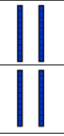

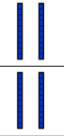









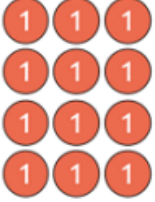
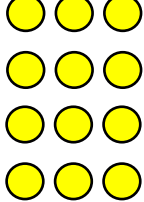


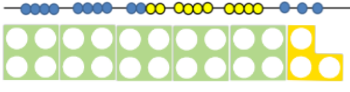
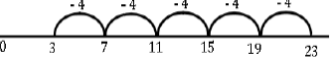
Year 1			
Outcome	Concrete	Pictorial	Abstract
Explore unequal groups	Explore unequal groups of familiar objects for children to understand equality. 	Images of unequal groups.  <i>There are some pencils. The pencils have been grouped. There are 3 groups.</i> <i>There are some footballs. The footballs have been grouped. There are 3 groups.</i>	
Multiplication as repeated addition, (link to counting in twos, fives and tens)	Use familiar objects to show repeated addition. 	Repeated images. How many wheels altogether? 	Number sentences: $2 + 2 + 2 = 6$ $5 + 5 + 5 + 5 = 20$
Represent repeated addition as an array	Explore arrays using a variety of familiar objects: 	Draw and interpret images of arrays as repeated addition. 	Write number sentences using repeated addition. $10 + 10 + 10 = 30$ $2 + 2 + 2 + 2 + 2 = 10$
<u>Division</u> Group small quantities (How many equal groups of...?)	Use familiar objects to put into 'groups of'. 	Explore images and draw circles to show 'groups of'. 	<u>Number sentences and problems.</u> Take 20 cubes. Complete the sentences. I can make ____ equal groups of 2 I can make ____ equal groups of 5 I can make ____ equal groups of 10
<u>Division</u> Share small quantities equally (How much does each part contain?)	Use familiar objects within a context to share equally between an amount. 	Explore images to share equally. 	<u>Number sentences and problems</u> Tim has 16 bananas. He shares them equally between two boxes. How many bananas are in each box?

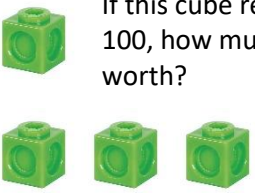
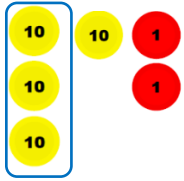
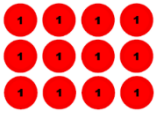
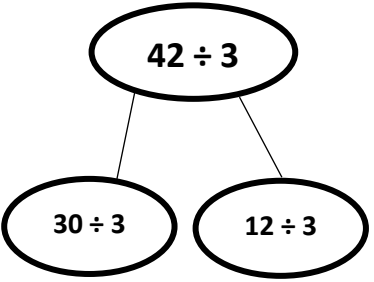
<p>Double numbers to 20</p>	<p>Use objects to explore the idea of having 'two lots of an amount.'</p>  <p>Single Double</p> 	<p>Tens frame</p>  <p>Numicon</p> <p>6 + 6 = 12</p>  <p>Images showing double</p> 	<p>$2 + 2 = 4$ Double 2 = 4 2 lots of 2 = 4</p>
<p>Halve numbers to 20. Find and name one half as two equal parts of a whole</p>	<p>Halve familiar objects using the idea of 'sharing into two equal groups.'</p> 	<p>Explore images relating to halving.</p>  <p>Explore equality.</p> 	<p>Whole = 6 Half = 3 Half of 6 = 3 $\frac{1}{2}$ of 6 = 3</p> <p>I have 6 sweets. I eat half. How many sweets did I eat?</p>
<p>Find and name one quarter as four equal parts of a whole (relating to dividing by 4)</p>	<p>Quarter familiar objects using the idea of 'sharing into four equal groups.'</p>  <p>whole $\frac{1}{4}$ 1 part out of 4</p> <p>Whole $\frac{1}{4}$</p>	<p>Explore images relating to quarters.</p> 	<p>Whole = 8 Quarter = 2 One quarter of 8 = 2 $\frac{1}{4}$ of 8 = 2</p>



Year 2			
Outcome	Concrete	Pictorial	Abstract
<p>Use repeated addition with the multiplication sign (x)</p> <p>Relate this to division with the division sign (\div)</p>	<p>Use familiar images to show repeated addition.</p>  	<p>Images to show repeated addition relating to multiplication</p>  <p>$5 + 5 + 5$ 3×5</p>  <p>$10 + 10 + 10 + 10$ 4×10</p> <p>Number lines</p>  	<p>Introduce the multiplication sign within number sentences:</p> <p>$3 \times 10 =$ $10 \times 3 =$ $30 = 10 \times \square$ $\square \times 3 = 30$</p>

<p>Use arrays to make or draw multiplications and division facts.</p>	<p><u>Numicon</u></p>  <p><u>Identify arrays in everyday objects</u></p> 	<p><u>Array images</u></p>  <p>3 lots of 2 $3 \times 2 = 6$</p> 	<p><u>Number sentences</u></p>  <p>$3 \times 4 = 12$ $12 \div 4 = 3$</p> <p><u>Missing numbers</u></p>  <p>$20 = \square \times 5$</p>
<p>Understand the commutative law.</p>	<p><u>Rotating arrays</u></p>  <p>$3 \times 2 = 6$</p>  <p>$2 \times 3 = 6$</p>	<p><u>Number lines</u></p>  <p>$3 \times 5 = 15$</p>  <p>$5 \times 3 = 15$</p>	<p><u>Number sentences</u></p> <p>$3 \times 5 = 15$ $5 \times 3 = 15$ $15 \div 3 = 5$ $15 \div 5 = 3$</p>


Year 3											
Outcome	Concrete	Pictorial	Abstract								
<p>Mental methods – distributive law (partition into smaller parts to multiply)</p>	<p><u>Dienes</u></p> <p>35×3</p>  <p>30×3 5×3</p> <p><u>Place value counters</u></p> <p>43×3</p> <table border="1" data-bbox="323 1279 635 1641"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table>	Tens	Ones							<p>Use of part/ part/ whole to show the partitions.</p> 	<p>Number sentences only:</p> <p>$35 \times 3 =$</p> <p>$30 \times 3 = 90$ $5 \times 3 = 15$</p> <p>$90 + 15 = 105$</p>
Tens	Ones										
											
											
											
<p>Mental methods – using known facts (2 digit)</p>	<p>Relate known times tables facts to larger numbers</p> <p><u>Place value counters</u></p> <p>Link ones to tens</p>  <p><u>Numicon</u></p> <p>Use the notion of what could one represent?</p>  <p>If each hole is worth 10, how much is this piece?</p>	<p><u>Finding patterns in numbers</u></p> <p>If we know that... $3 \times 2 = 6$... then we know...</p> <p>$30 \times 2 = 60$ $20 \times 3 = 60$ $60 \div 2 = 30$ $60 \div 3 = 20$</p>									

<p>Formal written methods – grid and short multiplication (TO x O)</p>	<p><u>Dienes</u></p> <table border="1" data-bbox="320 136 608 376"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table> <p>23 x 3</p> <p><u>Place value counters</u></p> <table border="1" data-bbox="320 461 635 824"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table> <p>43 x 3</p>	Tens	Ones							Tens	Ones							<p>Use arrays to help with commutativity of numbers. Use known facts to help to multiply larger numbers.</p> <p><u>Grid method</u></p> <p>Question: 24 x 5</p> <p>1) Partition the number into tens and ones</p> <p>20 x 5 4 x 5</p> <p>2) Draw this grid and fill in the boxes.</p> <table border="1" data-bbox="719 600 1086 797"> <tbody> <tr> <td>x</td> <td>20</td> <td>4</td> </tr> <tr> <td>5</td> <td>100</td> <td>20</td> </tr> </tbody> </table> <p>3) Add together the amounts: 100 + 20 = 120</p>	x	20	4	5	100	20	<p><u>Short multiplication</u></p> <p>No exchange:</p> 43×3 $\begin{array}{r} 43 \\ \times 3 \\ \hline 129 \end{array}$ <p>Exchange:</p> 37×3 $\begin{array}{r} 37 \\ \times 3 \\ \hline 111 \\ 2 \end{array}$
Tens	Ones																								
																									
																									
																									
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x	20	4																							
5	100	20																							
<p>Use arrays to show the link between multiplication and division.</p>	<p><u>Counters/ multi-link</u></p> <p>12 ÷ 4</p>  <p>4 rows of 3 12 ÷ 4 = 3</p>	<p><u>Represent through drawings</u></p> <p>12 ÷ 4</p>  <p>Again draw into columns of the divisor.</p>	<p>Write number sentences:</p> <p>3 x 4 = 12 4 x 3 = 12 12 ÷ 4 = 3 12 ÷ 3 = 4 12 = 3 x <input type="text"/> <input type="text"/> x 4 = 12</p>																						
<p>Begin to use formal written methods for division (not exchanging from tens to ones at this stage)</p>	<p>Use number beads to support calculations.</p> <p><u>Place value counters</u></p> <p>69 ÷ 3 =</p>  <p>Organise counters into columns of the divisor.</p>	<p>Extend this to a similar layout to short division.</p> <p>23</p> 	<p><u>Short division</u></p> <p>69 ÷ 3 =</p> $\begin{array}{r} 23 \\ 3 \overline{) 69} \end{array}$																						
<p>Begin to understand the concept of remainders</p>	<p>Using familiar resources</p> 	<p>Use repeated subtraction on a number line:</p> <p>23 ÷ 4 = 5 r3</p> 	<p>Begin to solve mentally:</p> <p>23 ÷ 4 = <input type="text"/></p> <p>Missing number problems:</p> <p><input type="text"/> ÷ 3 = 4 r1</p>																						

Year 4																				
Outcome	Concrete	Pictorial	Abstract																	
Mental methods – using known facts (3 digit)	<p><u>What could one represent?</u></p> <p>If this cube represents 100, how much is this worth?</p> 	<p><u>Finding patterns in numbers (3 digits)</u></p> <p>If we know that... $3 \times 2 = 6$... then we know...</p> <p>$30 \times 2 = 60$</p> <p>$300 \times 2 = 600$</p> <p>$30 \times 20 = 600$</p> <p>$600 \div 2 = 300$</p> <p>$600 \div 30 = 20$</p>																		
Mental methods – distributive law (partition into smaller parts to divide)	<p><u>Place value counters</u></p> <p>$42 \div 3 =$</p>  <p>3 tens $\div 3 = 1$ ten</p> <p>Extra ten exchange:</p>  <p>4 columns of 3</p> <p>Partition is $30 \div 3$ and $12 \div 3$</p>	<p>Use of part/ part/ whole to show the partitions.</p> 	<p>Number sentences only:</p> <p>$42 \div 3 =$</p> <p>$30 \div 3 = 10$</p> <p>$12 \div 3 = 4$</p> <p>$10 + 4 = 14$</p>																	
Formal written method – short multiplication (HTO x O)	<p>Begin with crossing one boundary.</p> <p>$243 \times 3 =$</p> <table border="1" data-bbox="319 1030 614 1265"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table>	Hundreds	Tens	Ones										<p><u>Short multiplication – one boundary</u></p> <p>126×3</p> $\begin{array}{r} 126 \\ \times \quad 3 \\ \hline 378 \\ 1 \end{array}$						
	Hundreds	Tens	Ones																	
<p>Then move onto crossing two boundaries.</p> <p>247×3</p> <table border="1" data-bbox="319 1377 630 1624"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table>	Hundreds	Tens	Ones										<p><u>Short multiplication – two boundaries</u></p> <p>247×3</p> $\begin{array}{r} 247 \\ \times \quad 3 \\ \hline 741 \\ 12 \end{array}$							
Hundreds	Tens	Ones																		
Formal written method – short division (TO \div O and HTO \div O)	<table border="1" data-bbox="319 1635 710 2072"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td>...becomes </td> <td></td> </tr> <tr> <td></td> <td>2 groups of 6 tens </td> <td></td> </tr> <tr> <td></td> <td></td> <td>...becomes </td> </tr> <tr> <td></td> <td></td> <td>3 groups of 6 ones </td> </tr> </tbody> </table> <p>$138 \div 6 =$</p>	Hundreds	Tens	Ones					...becomes 			2 groups of 6 tens 				...becomes 			3 groups of 6 ones 	<p><u>Short division</u></p> <p>$138 \div 3 =$</p> $\begin{array}{r} 23 \\ 3 \overline{) 138} \\ \underline{6} \\ 13 \\ \underline{9} \\ 48 \\ \underline{45} \\ 3 \end{array}$ <p>$432 \div 5$ becomes</p> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$ <p>Answer: 86 remainder 2</p>
Hundreds	Tens	Ones																		
	...becomes 																			
	2 groups of 6 tens 																			
		...becomes 																		
		3 groups of 6 ones 																		

Year 5			
Outcome	Concrete	Pictorial	Abstract
Short multiplication (ThHTO x O)	See Year 3 and 4 short multiplication.		Short method for ThHTO $\begin{array}{r} 2741 \\ \times \quad 6 \\ \hline 16446 \\ \quad 42 \end{array}$
Long multiplication (TO x TO then HTO x TO)	Children working at this level are expected to have secure knowledge of place value and will therefore work from pictorial and abstract.	Begin with expanded method to help with understanding of the method: $\begin{array}{r} 47 \\ \times 36 \\ \hline 42 \quad (7 \times 6) \\ 240 \quad (40 \times 6) \\ 210 \quad (7 \times 30) \\ \hline 1200 \quad (40 \times 30) \\ \hline 1692 \end{array}$	<u>Long multiplication</u> $\begin{array}{r} 47 \\ \times 36 \\ \hline 282 \\ \quad 4 \\ \hline 1410 \\ \quad 2 \\ \hline 1692 \end{array}$
Short division (ThHTO ÷ O)	Follow the guidance for short division in Year 4.	$2576 \div 3 =$	
		<u>Showing remainder as a whole:</u> $\begin{array}{r} 858 \text{ r } 2 \\ 3 \overline{) 2576} \\ \underline{3257} \\ 6 \\ \underline{6} \\ 0 \end{array}$ Answer: 858 remainder 2 <u>Showing remainder as a fraction:</u> $\begin{array}{r} 858 \text{ r } 2 \\ 3 \overline{) 2576} \\ \underline{3257} \\ 6 \\ \underline{6} \\ 0 \end{array}$ Answer: $858 \frac{2}{3}$	Showing remainder as decimals $\begin{array}{r} 858.66 \\ 3 \overline{) 2576.00} \\ \underline{3257} \\ 6 \\ \underline{6} \\ 0 \\ \underline{0} \\ 0 \end{array}$ Add place value digits to continue into decimals.
Mental methods – using known facts (link to division)	<u>What could one represent?</u>  If this cube represents 1000, how much is this worth? 		<u>Finding patterns in numbers (decimals)</u> If we know that... $3 \times 2 = 6$... then we know... $300 \times 2 = 600$ $30 \times 20 = 600$ $600 \div 2 = 300$ $600 \div 30 = 20$ $3 \times 0.2 = 0.6$ $0.3 \times 2 = 0.6$

Year 6

Outcome	Concrete	Pictorial	Abstract															
Use known facts	Consider what 'one' could represent, e.g. If this cube represents 0.1, how much is this worth? 	Finding patterns in numbers If we know that... $3 \times 2 = 6$... then we know... $30 \times 2 = 60$ $200 \times 3 = 600$ $0.3 \times 2 = 0.6$ $0.3 \times 20 = 6$ $600 \div 2 = 300$ $6000 \div 30 = 200$																
Long multiplication (ThHTO x TO)	Use the guidance from Year 5 for long multiplication applying it to ThHTO x TO.																	
Long division (ThHTO ÷ TO)	Children should be secure in place value at this stage.	<u>Expanded method:</u> $432 \div 15 =$ 1) Write down times tables for 15 x1 15 x6 90 x2 30 x7 105 x3 45 x8 120 x4 60 x9 135 x5 75 x10 150 2) Subtract suitable amounts. $\begin{array}{r} 28 \text{ r}12 \\ 15 \overline{)432} \\ \underline{300} \quad (15 \times 20) \\ 132 \\ \underline{120} \quad (15 \times 8) \\ 12 \quad \text{(remainder)} \end{array}$	<u>Long division</u> $432 \div 15 =$ Same as expanded except we bring down each digit rather than show place value. $\begin{array}{r} 28 \\ 15 \overline{)432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 12 \end{array}$ Bring down the 2 ones. 12 is the remainder Answer: 28 r 12															
Long division – interpreting the remainder as a decimal		Follow the same steps for expanded method.	<u>Long division</u> As with short division in Year 5, continue the place value into tenths by adding extra place holders (0). $\begin{array}{r} 28.8 \\ 15 \overline{)432.0} \\ \underline{30} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$ Add place holder after decimal point. Continue for up to 2dp.															
Short multiplication - decimals	Use place value counters <table border="1" data-bbox="316 1832 801 2076"> <thead> <tr> <th>Ones</th> <th>Tenths</th> <th>Hundredths</th> </tr> </thead> <tbody> <tr> <td>● ● ●</td> <td>● ● ● ●</td> <td>● ●</td> </tr> <tr> <td>● ● ●</td> <td>● ● ● ●</td> <td>● ●</td> </tr> <tr> <td>● ● ●</td> <td>● ● ● ●</td> <td>● ●</td> </tr> <tr> <td>● ● ●</td> <td>● ● ● ●</td> <td>● ●</td> </tr> </tbody> </table>	Ones	Tenths	Hundredths	● ● ●	● ● ● ●	● ●	● ● ●	● ● ● ●	● ●	● ● ●	● ● ● ●	● ●	● ● ●	● ● ● ●	● ●	4×3.42	<u>Short multiplication</u> 4×3.42 $\begin{array}{r} 3.42 \\ \times \quad 4 \\ \hline 13.68 \\ \hline 1 \end{array}$
Ones	Tenths	Hundredths																
● ● ●	● ● ● ●	● ●																
● ● ●	● ● ● ●	● ●																
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